

# On a stable solute gradient heated from below with prescribed temperature

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The splitting of a smooth gradient into mixed layers separated by thin interfaces is studied in a constant stable solute gradient heated uniformly from below by raising the temperature at its bottom to a prescribed value above the ambient. The phenomenon under study is characterized by the development of a first mixed layer from the bottom up to a certain height; it is shown that its instantaneous thickness is proportional to the thermal expansion coefficient and the instantaneous temperature difference between the bottom heated plate and the undisturbed far fluid and is inversely proportional to the solute gradient. A second layer starts its formation during the growth of the first one and persists above the bottom layer notwithstanding the advancing interface below. Further layer formation occurs, depending on the prescribed Rayleigh number of the system (which is based on the vertical temperature difference and the initial solute gradient). The secondary layers grow with time and the final thickness of all the layers (except the bottom one) is almost equal. Their average final thickness is found to depend mostly on the solute gradient and the fluid properties. A final quasi-steady state is reached in which the depth of the whole convective region is proportional to the expansion coefficient and the vertical temperature difference and is inversely proportional to the initial solute gradient.

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## 1. Introduction

Double diffusive layers may be formed in a fluid containing an initial stably solute gradient when it is heated uniformly from below. Turner (1968) was the first to study this phenomenon quantitatively, concentrating on the dynamics of the first mixed layer and the formation of the second layer above it. The thickness of the first layer was observed by Turner (1968) to increase with time as  $h_1(t) = 1.3(q_0)^{1/2}(g\phi_0)^{-1/2}t^{1/2}$ , where  $g$  is the gravitational acceleration,  $\phi_0 = -\beta dS_0/dZ$ ,  $\beta$  is the coefficient of solutal contraction,  $dS_0/dZ$  is the initial negative solute gradient, and  $q_0$  is the buoyancy flux defined as  $q_0 = \alpha g Q / \rho_0 c_p$ , where  $\alpha$  is the coefficient of thermal expansion,  $Q$  is the applied heat flux,  $\rho_0$  is a reference density and  $c_p$  is the specific heat. The first layer stopped growing at a certain height owing to the formation of a second mixed layer above it.

The problem was further studied experimentally and theoretically by Huppert & Linden (1979). They extended Turner's work to the formation of multiple layers and the transport between them. The thickness of the secondary layers was significantly smaller than that of the first layer, and all of them were nearly uniform in depth. Their average thickness was found to be  $\bar{h}_s = 51(k_T^2/g\phi_0)^{1/2}$ , where  $k_T$  is the molecular

thermal diffusivity. This scale for the average thickness of the secondary layers is independent of the heat flux and depends only on the solute gradient (stabilizing component) and the fluid properties. The height of the advancing front for the whole layered system was found by Huppert & Linden (1979) to be represented by  $h_T = 2.404(q_0)^{1/3}(g\phi_0)^{-1/2}t^{2/3}$ .

Recently, Fernando (1987) showed that the first mixed layer initially grows owing to the action of eddies that rise from the bottom heated plate. At a certain moment during this growth a second layer starts to form but it is entrained by the advancing front until the vertical kinetic and potential energies of the eddies are of the same order. At this state the first layer almost stops growing and a second quasi-stationary layer bounded by stable interfaces can exist above it.

A related situation of interest is that of a constant stable solute gradient, heated from below by raising the bottom temperature to a prescribed value above the ambient. Lewis, Incropera & Viskanta (1982) studied this problem experimentally in a relatively small tank and observed only one secondary layer whose existence was temporary. This behaviour was attributed by Lewis *et al.* (1982) to the isothermal bottom heating and to the decrease in time of vertical heat transfer through the solution.

In this work we investigate a system that has a constant stable solute gradient, initially at uniform temperature. The bottom temperature is raised to a prescribed value above the ambient according to some function of time. The aim of the present work is to study the behaviour of the layered system under a temperature boundary condition and in particular to determine the typical scales of the layers and the whole convective region.

The research is basically experimental but it includes some theoretical considerations concerning the growth of the bottom mixed layer (§2). The experimental facilities are described in §3 and the experimental results are described and discussed in §4, along with the comparisons with previous works. Section 5 summarizes the paper with the concluding remarks. The material of this paper can be found in more detail in Kerpel (1989).

## 2. Theoretical considerations

### 2.1. *Introductory notes*

Laboratory experiments of a linear stable solute gradient heated from below, as reviewed by Tsinober (1988), show that there are two kinds of layers, one represented by the bottom layer and the other by the layers formed on top of the first one. The vertical scale for the bottom layer was found by Turner (1968) to depend on the applied heat flux, the solute gradient and the time, as shown in §1. On the other hand, the scale for the secondary layers was found by Huppert & Linden (1979) (§1) to depend on the solute gradient and the fluid properties only, and will be treated in more detail in §2.3 below.

### 2.2. *A model for the first convective layer*

The present model is a modification of Turner's (1968) model, and it considers the growth of the first layer when the bottom temperature  $T_b$  is raised above the temperature of the undisturbed far fluid  $T_\infty$  according to some function of time (Tanny, Kerpel & Tsinober 1989).

In the model we incorporate Turner's (1968) assumptions that (i) the growing first layer has homogeneous solute concentration over its depth and (ii) the diffusion of

salt through the advancing front is negligible. The third assumption made by Turner that diffusion of heat through the interface is negligible is not incorporated here, and instead we assume that the temperature step across the interface  $\Delta T_i(t)$  is some constant fraction of the overall vertical temperature difference  $\Delta T_b(t)$ :

$$\Delta T_i(t) = C_0 \Delta T_b(t), \quad (1)$$

where

$$\Delta T_b(t) = T_b(t) - T_\infty. \quad (2)$$

The experimental results reported later (§4.1) show that  $C_0$  is constant once the bottom mixed layer has been formed. Therefore the analysis will be continued using this assumption.

As in Turner's model, it is further assumed that the interface is just unstable, implying that the density steps due to salinity and temperature at the interface are equal and opposite:

$$\alpha \Delta T_i(t) = -\beta \Delta S_i(t) \quad (3)$$

where

$$\Delta S_i(t) = \frac{1}{2} h_1(t) \frac{dS_0}{dZ}. \quad (4)$$

Although the density interface should be considered as marginally stable it is assumed here as neutrally stable since the difference between the two states is small (about 10%).

From (1), (3) and (4) one can obtain  $h_1(t)$  as

$$h_1(t) = 2C_0 \left( \frac{\alpha \Delta T_b(t)}{\phi_0} \right) = 2C_0 \eta(t) \quad (5)$$

where  $\eta$  is the lengthscale  $\eta = \alpha \Delta T / \phi_0$ , first defined by Chen, Briggs & Wirtz (1971) in another context.

### 2.3. *The thickness of the secondary layers*

Huppert & Linden (1979) observed that all secondary layers had nearly the same thickness, the average of which was found to be

$$\bar{h}_s = 51(k_T^2/g\phi_0)^{\frac{1}{4}}, \quad (6)$$

independent of the applied heat flux.

A similar lengthscale was suggested by Tanny (1988) and Tanny & Tsinober (1988) for sidewall heating of a solute gradient. Their scale was independent of the lateral temperature difference and also depended on the solute gradient and the fluid properties:

$$\xi = \left( \frac{\nu k_S}{g\phi_0} \right)^{\frac{1}{4}}, \quad (7)$$

where  $\nu$  is the kinematic viscosity and  $k_S$  is the solute diffusivity.

With the use of the lengthscale  $\xi$  and for  $\tau = k_S/k_T = 1/80$  (for heat and salt), (6) becomes

$$\bar{h}_s = 98.7\xi. \quad (8)$$

The main difference between this expression and that obtained by Huppert & Linden (1979) is that  $\bar{h}_s$  in (8) is a function of the solute diffusivity  $k_S$  instead of being a function of the thermal diffusivity  $k_T$ .

In order to check the dependence of  $\bar{h}_s$  on  $k_s$ , experiments with heat and different solutes were performed. The solutes are salt, sugar and polyethylene glycol; their respective molecular diffusivities in water are:  $1.5 \times 10^{-5}$  for salt,  $0.5 \times 10^{-5}$  for sugar and  $5.65 \times 10^{-7}$  cm<sup>2</sup>/s for polyethylene glycol.

In conclusion it is noteworthy that there are two different scales relevant to the problem in question,  $\eta$  and  $\xi$  defined earlier.

### 3. Experimental apparatus and techniques

The experiments were performed in a tall glass tank  $40 \times 40 \times 100$  cm high. The base of the tank consisted of a 4.7 mm brass plate below which were attached 10 strip heating resistors operated by a variable power source. The four walls of the tank were insulated with special transparent insulation units, similar to those used by Tanny (1988) and Tanny & Tsinober (1988). Each unit was a sandwich of three thin Plexiglas plates (1.5 mm), separated by 4.5 mm air gaps. This special insulator allowed flow visualization during the course of the experiment, and prevented the possible formation of double-diffusive layers due to sidewall cooling. The free surface of the fluid was also insulated by a floating plate of plastic insulator.

The heated-surface temperature was measured by nine copper-constantan thermocouples (type T) with accuracy of  $\pm 0.3$  °C. One thermocouple was located up in the undisturbed fluid in order to measure the vertical temperature difference between the bottom heated plate and the ambient fluid. The vertical temperature difference was measured and controlled by a system consisting of a Data Logger and a VAX-750 computer. The temperature of the bottom plate was allowed to rise with time according to  $\Delta T_b = \Delta T_{b_{\max}} [1 - \exp(-t/t_0)]$ , where  $\Delta T_{b_{\max}}$  is the prescribed vertical temperature difference and  $t_0$  is the time constant of heating. In two of the experiments the temperature was raised linearly with time according to the expression  $\Delta T_b = ct$ , where  $c$  is a fixed constant. These experiments are hereafter referred to as linear heating experiments.

The linear solute gradient was set up in the tank by the standard 'double-bucket' method (Oster 1965) using salt, sugar and polyethylene glycol solutions. The thermophysical properties for the solutions were taken from Barber (1966), Ruddick & Shirlcliffe (1979) and Brandrup & Immergut (1966). The kinematic viscosity of polyethylene glycol solutions is about 5 times larger than that of salt or sugar solutions, resulting in a Prandtl number ( $Pr = \nu/k_T$ ) of about 35 for the experiments with polyethylene glycol.

A total of 41 experiments with various physical parameters were carried out. For the salt experiments the ambient density gradient was varied in the range  $\phi_0 = 1.62 \times 10^{-4}$ – $5.9 \times 10^{-4}$  cm<sup>-1</sup> while the prescribed temperature-difference variation was  $\Delta T_{b_{\max}} = 3$ – $19$  °C. For the sugar experiments the ambient density gradient was varied in the range  $\phi_0 = 1.45 \times 10^{-4}$ – $2.79 \times 10^{-4}$  cm<sup>-1</sup> and the prescribed temperature-difference variation was  $\Delta T_{b_{\max}} = 2.4$ – $9.5$  °C. For the polyethylene glycol experiments  $\phi_0$  was either  $1.56 \times 10^{-4}$  or  $2.94 \times 10^{-4}$  cm<sup>-1</sup> while the maximum temperature difference was equal to 6 °C. The heating time constant was chosen as  $t_0 = 800$  s for all the experiments.

The thickness of the layers was measured simultaneously by the tracer dye and the shadowgraph technique. The fluid was tagged with vertical lines of fluorescent dye (Rhodamine B) produced by dye particles dropped from the top of the tank. In addition, the thin fluid layer which is in contact with the bottom plate was dyed by fluorescein to facilitate the identification of the bottom mixed layer. The dyed fluid

was illuminated by a 0.5 cm thick sheet of white light. The shadowgraph was produced by a 3 W argon laser beam, expanded onto a large concentrating mirror, 28 cm in diameter.

## 4. Experimental results

### 4.1. The evolution of the first mixed layer

The non-dimensional instantaneous thickness of the first mixed layer  $h_1/\eta$  is plotted in figure 1 as a function of the instantaneous  $Ra_\eta$  ( $= g\alpha\Delta T_b \eta^3/\nu k_T$ ) for ten typical experiments with exponential and linear heating, noting that similar results were obtained in all the others. It is observed that  $h_1/\eta$  is independent of  $Ra_\eta$ , in a range over more than five orders of magnitude. The average instantaneous height of the first mixed layer is

$$h_1 = (0.937 \pm 0.168) \eta, \quad (9)$$

The result above supports the model assumption (§2.2) that  $C_0$  is a constant; its average experimental value is  $C_0 = 0.47 \pm 0.08$ .

It is known that the solute gradient is zero at the bottom owing to the zero flux condition, and the region with zero gradient grows with time. The thickness of this region at the start of heating was about 5–8 mm. This region may influence the instability of the initial solute gradient due to heating, but certainly not the subsequent behaviour of the much thicker bottom mixed layer once it has been formed.

For comparison, data extracted from Turner (1968), Fernando (1987 and private communication) and Lewis *et al.* (1982) are also included in figure 1. The excellent agreement between our results and those of Turner (1968) and Fernando (1987) indicates that for both boundary conditions (constant heat flux and prescribed temperature) the instantaneous thickness of the bottom layer is directly proportional to  $\eta$  according to (9).

A complete comparison with the results of Lewis *et al.* (1982) is rather difficult because they heated the bottom plate very fast (in within 120 s) and they do not provide enough data about the growth of the bottom layer during this period. The single data point on figure 1 is from the moment at which the bottom reached the prescribed  $\Delta T_b$  (and  $Ra_\eta$ ) and is in good agreement with our results. The further growth of the bottom mixed layer at this prescribed  $Ra_\eta$ , as reported by Lewis *et al.* (their figure 14), suggests that when the bottom temperature is raised very fast compared with the growth rate of the first layer,  $C_0$  does not remain constant throughout this growth process.

### 4.2. The thickness of the secondary layers

In the present experiments it was observed that the second layer is formed during the growth of the first one, and persists above it notwithstanding the advancing interface below (Tanny *et al.* 1989). The thickness of the second layer increased with time up to a final value and a third layer was formed during the growth of the first one. Similarly, subsequent secondary layers were formed depending on the prescribed Rayleigh number.

The final thickness of all the secondary layers was almost the same (in agreement with the observations of Huppert & Linden 1979). This thickness is the maximum one attained by a secondary layer during its growth (excluding merging events as discussed below). In figure 2 the final non-dimensional average thickness of the

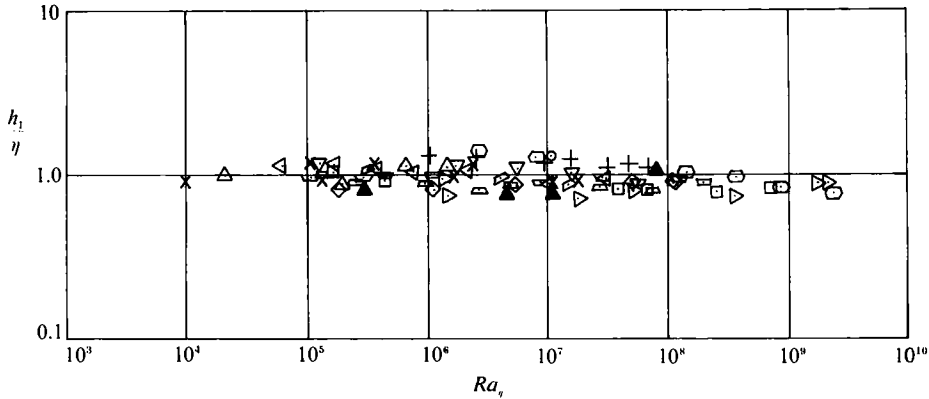


FIGURE 1. The non-dimensional instantaneous thickness of the first convective layer  $h_1/\eta$  as a function of the instantaneous Rayleigh number,  $Ra_\eta$ ;  $\blacktriangle$ , Polyethylene glycol;  $\diamond$ , sugar;  $\triangleright$ , linear heating (salt);  $\square$ , Fernando (1987);  $+$ , Turner (1968);  $\odot$ , Lewis *et al.* (1982); the other symbols correspond to salt experiments.

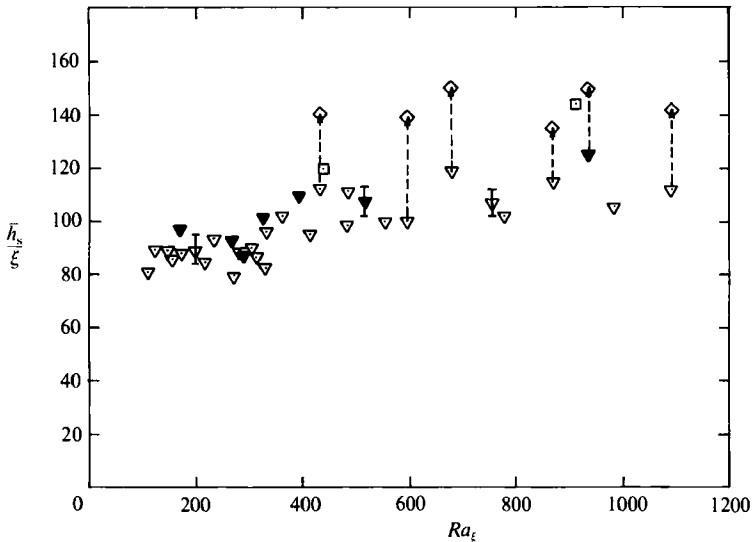


FIGURE 2. The average non-dimensional final thickness of the secondary layers  $\bar{h}_s/\xi$  as a function of the prescribed  $Ra_\xi$ :  $\nabla$ , salt;  $\blacktriangledown$ , sugar;  $\square$ , polyethylene glycol;  $\diamond$ , after merging.

secondary layers  $\bar{h}_s/\xi$  is plotted against the prescribed  $Ra_\xi$  for each experiment.  $Ra_\xi$  is defined as  $Ra_\xi = g\alpha\Delta T_b \xi^3 / \nu k_s$  and is related to  $Ra_\eta$  by  $Ra_\xi = (Ra_\eta/\tau)^{1/3}$ . Excluding the merging events (indicated by vertical arrows), the final average thickness of the secondary layers is almost constant, with a weak dependence on  $Ra_\xi$ . For  $100 < Ra_\xi < 450$  the average thickness of secondary layers  $\bar{h}_s$  is equal to  $(89.53 \pm 6.14)\xi$  while for larger  $Ra_\xi$  (between 450 and 1100) the average value of  $\bar{h}_s$  is equal to  $(109.33 \pm 7.9)\xi$ .

The value of  $\bar{h}_s/\xi$  obtained from figure 2 before merging takes place is similar to the value deduced from the scale found by Huppert & Linden (1979) for heat flux applied to the bottom plate ((8) in §2.3). It should be noted that their experiments

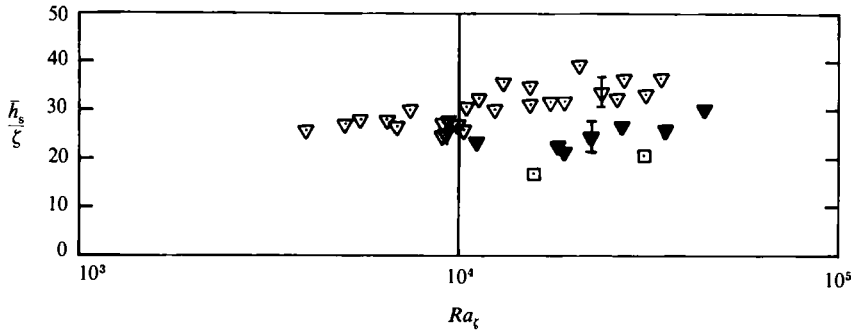


FIGURE 3. The average non-dimensional final thickness of secondary layers  $\bar{h}_s/\zeta$  as a function of the prescribed  $Ra_\zeta$ :  $\nabla$ , salt;  $\blacktriangledown$ , sugar;  $\square$ , polyethylene glycol.

were performed for large Rayleigh numbers. The above result indicates that the average final thickness of the secondary layers is a function of the solute gradient and the fluid properties.

Merging events are represented in figure 2 by vertical arrows, which indicate the final average thickness of secondary layers after merging occurs. Merging between adjacent secondary layers was only observed in experiments with high  $Ra_\zeta$  ( $> 450$ ) and resulted in an increase of the average thickness of the secondary layers to  $\bar{h}_s = (142.76 \pm 6.86) \zeta$ .

To verify that  $k_S$  should appear in the scale  $\xi$ , the results of figure 2 were plotted again in figure 3 but with  $\bar{h}_s$  normalized by  $\zeta$ , a lengthscale similar to  $\xi$  but based on  $k_T$ :  $\zeta = (\nu k_T / g \phi_0)^{1/2}$ , and  $Ra_\zeta = g \alpha \Delta T_b \zeta^3 / \nu k_S$ . It is noteworthy that in spite of the large difference between the solute diffusivity of sugar and polyethylene glycol (about one order of magnitude), the results for these two substances are very close owing to the larger viscosity of polyethylene glycol solutions.

It is shown that the data of the nondimensional thickness for the three different solutes in figure 2 collapse much better than in figure 3 where a systematic difference is observed between the results for salt and those for sugar and polyethylene glycol. In particular for  $Ra_\zeta > 10^4$  (figure 3) the average thickness of secondary layers is  $\bar{h}_s = (33.21 \pm 2.80) \zeta$  for the salt experiments, while for the sugar and polyethylene glycol experiments it is lower by about 30%,  $\bar{h}_s = (23.60 \pm 4.18) \zeta$ . Although the data are rather scattered the above result indicates that the scale for the average final thickness of secondary layers  $\bar{h}_s$  should be based on  $k_S$  and not on  $k_T$ .

#### 4.3. *The depth of the whole convective region*

The depth of the whole convective region  $h_T$  was measured during the experiment from the bottom plate up to the uppermost interface. The final depth of this region is the maximum one attained by the system of layers, and it was measured after a long period of time (at least 3–5 h and in some cases even 10 h), during which it remained unchanged. It is noteworthy that merging events (discussed in §4.2) hardly influence the final depth of the whole convective region.

The final depth of the whole convective region is shown in figure 4 as a function of the prescribed Rayleigh number  $Ra_\eta$  for 36 experiments. The average value obtained from this figure is

$$h_T = (2.0 \pm 0.3) \eta. \tag{10}$$

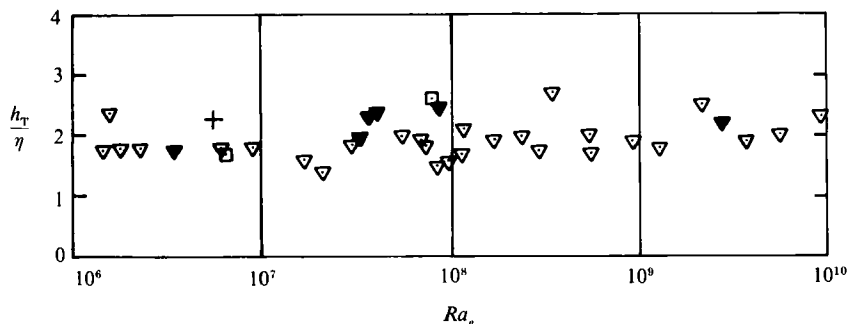


FIGURE 4. The final non-dimensional thickness of the whole convective region  $h_T/\eta$  as a function of the final  $Ra_\eta$ :  $\nabla$ , salt;  $\blacktriangledown$ , sugar;  $\square$ , polyethylene glycol; +, Lewis *et al.* (1982).

A data point extracted from Lewis *et al.* (1982) is also plotted in figure 4 and a good agreement between their results and the present ones is observed.

Since the thickness of the bottom layer is equal to  $\eta$  (figure 1), the above result indicates that the total thickness of all the secondary layers above it is also equal to  $\eta$ . The results of Turner (1968) and Huppert & Linden (1979) also show that the depth of the whole convective region is approximately twice the thickness of the bottom mixed layer (§1).

## 5. Concluding remarks

In the present experimental research a constant stable solute gradient was destabilized by raising the temperature of the bottom to a prescribed value above the ambient according to some function of time. A bottom mixed layer first develops and it is shown that its instantaneous thickness is proportional to  $\eta$  ( $= \alpha \Delta T_b / \phi_0$ ). If the bottom temperature is raised very fast compared with the growth of the first layer (Lewis *et al.* 1982), this layer grows further at the prescribed  $Ra_\eta$ , and  $C_0$  (equation (1)) does not remain constant.

Secondary layers are formed above this mixed layer; all of them are of almost equal final thickness whose average value is found to be proportional to  $\xi = (\nu k_s / g \phi_0)^{1/2}$ . A weak dependence on  $Ra_\xi$  is shown. For  $Ra_\xi$  between 100–450,  $\bar{h}_s$  is equal to  $89.5\xi$ , while for larger  $Ra_\xi$  (between 450–1100)  $\bar{h}_s$  is equal to  $109.3\xi$ . The average thickness of the secondary layers grows to  $142.8\xi$  once merging between adjacent layers takes place.

A final quasi-steady state is reached in all the experiments in which the final depth of the whole convective region is  $h_T = 2\eta$ .

The large value of  $Pr$  associated with the experiments with polyethylene glycol solution does not seem to have any significant influence on the measured thickness of the first and secondary layers.

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